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which is not even approximately the case. Large values of n are excluded by other considerations.

35. Hence it is no easier to detect optical distortion in the linear terms than in those of the second order, and we must regard the discrepancies as due to accidental causes—perhaps errors of copying—in the same way as the second order terms.

36. Thus we may feel some confidence that the optical distortion over a considerable field is small; and that accurate positions of stars may be obtained from these large plates if proper precautions be taken. I do not think, however, that the present line of examination is the best way of arriving at precise information about a small optical distortion; the method of trails, mentioned by Captain Hills, R.E., at the April meeting of the Society, seems better.

In another paper I have examined the necessary formulæ of reduction, and find them very simple; and I think the method will prove to be a very easy and direct way of measuring the optical distortion of a lens. The foregoing investigations will, however, serve to show how two plates may be compared when we have no independent information as to the optical distortion.

Conclusions of the Present Paper.

1. The errors of copying due to curvature of plate and oblique illumination, suggested in the last paper, were reproduced by direct experiment. The numerical value found was smaller than was expected, though this is capable of explanation. (§§ 1–7.)

2. A third region, including stars up to 4° from the centre of the Arequipa plate, was compared with an Oxford plate, and the terms of the second order gave no evidence of optical distortion. (§§ 8–15.)

3. The terms of the first order for all three plates dealt with in this paper and the last were examined, and gave no evidence of optical distortion. (§§ 16–35.)

4. Hence it seems possible to obtain good results over a region of (say) 5° × 5° with a photographic doublet.

5. Further experiments are, however, desirable by the method of star-trails, as indicated elsewhere.

On the Curvature of Star-trails on a Photographic Plate as a Means of Investigating Optical Distortion. By H. H. Turner, M.A., F.R.S., Savilian Professor.

1. In the discussion on my paper on optical distortion, read at the April meeting of the Society, Captain Hills, R.E., mentioned a simple method of investigating the distortion on a wide-angle photograph, which he had used in practice—viz. to take a series of star-trails on the plate and to compare their curvature with

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the theoretical curvature. If the field be not too large, the theoretical curvature is easily calculated from the formula

$$y = x^2 \times \tfrac{1}{2} \tan \delta,$$

where x y are expressed in circular measure.

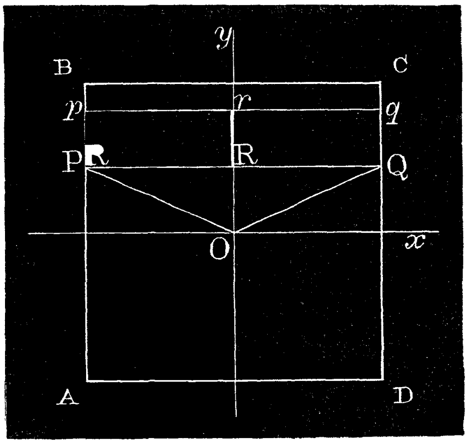
But with large fields it becomes necessary to employ a more accurate formula and to correct for refraction. If spherical coordinates are employed such formulæ are troublesome, but with rectangular coordinates they can be put into a simple form.

2. Let us consider first how optical distortion would affect the curvature of trails in different parts of the plate. We shall suppose the distortion to be a displacement of points along the radius from the centre of the field, and varying as some power of the radius. To fix the ideas, let us suppose it varies as the cube of the radius, so that

$$\Delta r = \mathrm{A}r^3,$$

and thus

$$\Delta x = \mathrm{A}r^2 x, \qquad \Delta y = \mathrm{A}r^2 y.$$



Let A B C D be a plate centre O, axes x and y parallel and perpendicular to the trails, which are nearly straight.

Let P R Q be a trail; and let

$$\mathrm{QR} = \mathrm{RP} = x, \quad \mathrm{OR} = y.$$

Then distortion, which increases any ordinate y by $\Delta y \equiv \mathrm{A}r^2 y$, elevates

$$\text{P and Q by } \mathrm{A}\,(x^2 + y^2)\,y, \text{ and R by } \mathrm{A}y^3 \text{ only},$$

so that the depth of R below P Q is increased by

$$\mathrm{A}x^2 y \text{ (approximately)}.$$

If we take another trail, prq, higher up the plate, with ordinate y', the effect of distortion on its curvature will be

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Ax^2y'; and thus the distortion affects the curvatures more and more as the trails travel up the plate in direct ratio to the distance.

Had we selected another law of distortion, say $\Delta r = Ar^5$, we should have had for the effect on curvature

$$\begin{aligned} &A\,(x^2+y^2)^2y - Ay^5 \\ &= A\,(x^4+2x^2y^2)y, \end{aligned}$$

which varies more rapidly than the first power of y at a distance from the centre. If we take even powers of r for Δr, such as

$$\Delta r = Ar^2 \text{ or } Ar^4,$$

we cannot express the effect of curvature as a finite series of powers of x and y; but the general run of the effect will be intermediate in character between that of two odd powers.

3. Thus we must be prepared to deal with trails in all parts of the plate—*i.e.* x and y may have values corresponding to the corners of the plate. We must now settle how big a plate we are going to measure. Suppose the plate something over 11° square, so that the coordinates of the corners, measured from the centre, are ±0·1 in circular measure. The maximum value of successive powers and products of x and y is then shown in the following table:—

Circular Measure.	Arc	Circular Measure.	Arc.
	′		″
x = 0·1	345·0	x^4 = ·0001	20·7
x^2 = 0·01	34·5	x^5 = ·00001	2·1
x^3 = 0·001	3·5	x^6 = ·000001	·21
		x^7 = ·0000001	·02

On such plates we may certainly reject x^7 and higher powers without any loss of accuracy—perhaps even x^6; but the computations will be carried out as far as x^6.

Plates larger (in angular field) than this have been used; and we may readily estimate the effect of going beyond this limit. Suppose, for example, that the plate is four times this area—*i.e.* 23° × 23°. Then x^7 would represent in the corners 2″·6. Now this is not a *very* large error to make as a maximum in a plate with this immense field, for unless the plate is of very large actual size, the scale must be comparatively small. For instance, the plate is unlikely to be larger than 23 inches × 23 inches, or 1 inch to the degree, one-third the scale of the Astrographic Catalogue plates; so that neglecting 2″·6 in the corners is equivalent to neglecting 0″·9 in the corners of Catalogue plates. Thus, though the formulæ developed in the following paragraphs are strictly accurate only for plates not larger than 11° × 11°, still they are probably accurate enough for *any* plate likely to be taken. If exceptionally

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large plates are taken in the future, the calculations can easily be extended for these exceptional cases.

4. *Theoretical Curvature without Refraction.*—The standard coordinates of a star on a plate are given by the following formulæ (*Monthly Notices*, liv. p. 17):—

$$\xi = \tan(\alpha - \mathrm{A}) \sin q \sec(\mathrm{P} - q), \quad \eta = \tan(\mathrm{P} - q),$$

where

$$\tan q = \tan p \cos(\alpha - \mathrm{A}),$$

α and p being the R.A. and N.P.D. of the star,
A and P ,, ,, ,, plate centre.

To get the relation between ξ and η for a trail, we must eliminate $(\alpha - \mathrm{A})$ between these two equations. The result may be set down without taking up space with the working; and since we shall be dealing with stars near the Equator, it will be more convenient to substitute $90° - \delta$ and $90° - \mathrm{D}$ for p and P respectively. The equation to a "trail" is thus found to be

$$\xi^2 \sin^2 \delta = [\eta \cos(\delta - \mathrm{D}) - \sin(\delta - \mathrm{D})]\, [\eta \cos(\delta + \mathrm{D}) + \sin(\delta + \mathrm{D})].$$

When

$$\xi = 0, \quad \eta = \tan(\delta - \mathrm{D}), \text{ or } -\tan(\delta + \mathrm{D}).$$

The case with which we are concerned is the former, the latter root referring to the lower culmination of the star in the meridian of the plate.

Put $\eta = \tan(\delta - \mathrm{D}) + z$, where z is small.

The equation becomes

$$\xi^2 \sin^2 \delta = z \sin 2\delta + z^2 \cos(\delta - \mathrm{D}) \cos(\delta + \mathrm{D}),$$

or

$$z = \tfrac{1}{2} \tan \delta \,.\, \xi^2 - z^2 \cos(\delta - \mathrm{D}) \cos(\delta + \mathrm{D}) \operatorname{cosec} 2\delta.$$

5. At first sight it would seem that the second term on the right is large when δ is small; but $z^2 \operatorname{cosec} 2\delta$ is always small, and hence the term is small. Neglecting it gives the usual expression for curvature quoted in §1.

If we put

$$z \cot \delta = u,$$

then

$$\begin{aligned} u &= \tfrac{1}{2}[\xi^2 - u^2 \cos(\delta - \mathrm{D}) \cos(\delta + \mathrm{D}) \sec^2 \delta] \\ &= \tfrac{1}{2}[\xi^2 - u^2 (1 - \sin^2 \mathrm{D} \sec^2 \delta)]. \end{aligned}$$

6. Thus when D = 0, if we put u_0 for the value of u,

$$u_0 = \tfrac{1}{2}[\xi^2 - u_0^2]$$

or

$$u_0^2 + 2u_0 - \xi^2 = 0$$

$$\begin{aligned} \therefore u_0 &= (1 + \xi^2)^{\frac{1}{2}} - 1 \\ &= \tfrac{1}{2}\xi^2 - \tfrac{1}{8}\xi^4 + \tfrac{1}{16}\xi^6. \end{aligned}$$

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Thus

$$z=[\tfrac{1}{2}\xi^2-\tfrac{1}{8}\xi^4+\tfrac{1}{16}\xi^6]\tan\delta.$$

Referring to the table in §3, we see that the maximum value of $\frac{1}{16}\xi^6$ is $0''\cdot01$, and we may therefore reject this term. Since δ is small the term $\frac{1}{8}\xi^4\tan\delta$ is also small, and the formula

$$z=\tfrac{1}{2}\xi^2\tan\delta$$

is very accurate even for large plates.

7. When D is not zero, but at the same time D and δ are less than 45°, then putting

$$\kappa=1-\sin^2 D\sec^2\delta,$$

we have

$$\kappa u^2+2u-\xi^2=0,$$

$$u=\frac{1}{\kappa}\{\sqrt{1+\kappa\xi^2}-1\},$$

$$=\tfrac{1}{2}\xi^2-\frac{\kappa}{8}\xi^4+\frac{\kappa^2}{16}\xi^6;$$

$$z=\left[\tfrac{1}{2}\xi^2-\frac{\kappa}{8}\xi^4+\frac{\kappa^2}{16}\xi^6\right]\tan\delta.$$

Since κ^2 is less than unity, the term $\frac{\kappa^2}{16}\xi^6$ can be rejected still more than before; and thus

$$z=\left(\tfrac{1}{2}\xi^2-\frac{\kappa}{8}\xi^4\right)\tan\delta$$

is an accurate formula for trails when δ and D lie between 0° and 45°.

8. But in experiments on optical distortion we do not need the absolute curvature of a trail, only the *difference* of curvatures between trails in the middle of the plate and elsewhere; and in many instances we do not require any formula at all. There are two distinct cases likely to occur in practice.

(A) When trails of the same star are taken in different portions of the plate; *i.e.* δ remains constant, and D varies from $\delta+5^\circ\cdot7$ to $\delta-5^\circ\cdot7$.

(B) When the plate centre is kept fixed and different stars allowed to trail over it; here D is kept fixed and δ varies from $D+5^\circ\cdot7$ to $D-5^\circ\cdot7$.

9. Take Case A first. Let $D=\delta+\eta$. Then $\eta<5^\circ\cdot7$ and

$$\kappa=1-\sin^2(\delta+\eta)\sec^2\delta$$

$$=1-\tan^2\delta-2\eta\tan\delta$$

$$=\kappa_0-2\eta\tan\delta,\text{ say.}$$

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Let z_0 refer to a trail through the plate centre. Then

$$z_0 = \left(\tfrac{1}{2}\xi^2 - \frac{\kappa_0}{8}\xi^4\right)\tan\delta$$

$$z = \left(\tfrac{1}{2}\xi^2 - \frac{\kappa_0 - 2\eta\tan\delta}{8}\xi^4\right)\tan\delta;$$

$$\therefore\ z - z_0 = +\tfrac{1}{4}\eta\,.\tan^2\delta\,.\,\xi^4.$$

10. When $\delta = 45°$ the maximum value of this expression, *i.e.* its value when $\xi = 0{\cdot}1$, and $\eta = 0{\cdot}1$ is 0″·5 (see § 3); and thus for work in which we reject quantities less than 1″·0 we may consider trails of the same star to have the same theoretical curvature all over the plate. For more accurate work we may use the following small table :—

Table of differences of curvature for trails of the same star in different parts of a plate.

If the trail be m degrees *south* of the plate centre, and $2n$ degrees long from end to end, and if the height of the ends above the middle be measured in seconds of arc and compared with a similar result for a trail of the same star through the plate centre, the excess of the former above the latter is $mn^4 \times {\cdot}00001$ times the quantity tabulated below.

Decl. of Star.	Excess.	Decl. of Star.	Excess.
°	″	°	″
10	+0·3	37·5	+4·8
20	+0·8	40·0	+6·0
30	+2·3	42·5	+7·1
35	+4·2	45·0	+8·4

11. (B) If we expose the same plate to a series of stars of different declinations, without moving the plate centre, the calculation is not so simple. For the curvatures of two stars declinations δ_1 and δ_2 will differ by a quantity of a different order, viz. :

$$\tfrac{1}{2}\,\xi^2\,(\tan\delta_1 - \tan\delta_2).$$

But using the table of Case A to reduce the curvature to the value it would have had if the trail had gone through the plate centre, we can now form tables for stars of different declinations giving the curvature of their trails when central on a plate. In fact we have to tabulate z_0 of the preceding case, viz. :

$$z_0 = \tfrac{1}{2}\tan\delta\,.\,\xi^2 - \tfrac{1}{8}(\tan\delta - \tan^3\delta)\,\xi^4.$$

The maximum value of the coefficient of ξ^4, which vanishes when $\delta = 0°$ and $\delta = 45°$, occurs when $3\tan{}^2\delta = 1$, *i.e.* for $\delta = 30°$; and its value is then 0·048, the effect on z_0 being 1″·0.

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12. Thus if we are neglecting quantities less than 1″, we may neglect this term altogether; *i.e.* combining this result with that of the last paragraph we may say that

The formula $z = \frac{1}{2} \tan \delta \,.\, \xi^2$ *gives the curvature of all star trails on plates not larger than* 11° × 11° *wherever they may be on the plate, when* δ *lies between* 0° *and* 45° *with errors less than* 1″.

13. For more accurate work it is not difficult or troublesome to calculate the two coefficients of § 11; but a small table of $\frac{1}{8}(\tan \delta - \tan^3 \delta)$ will help, and then we can include both Cases A and B under one general formula, as follows:—

Let δ be the declination of a star and let its trail be $2n$ degrees long, and m degrees south of the plate centre. The height of the ends above the middle in seconds of arc is given by

$$z = 31''{\cdot}416 \,.\, n^2 \,.\, \tan \delta + (mn^4 \,.\, \mathrm{A} \,.\, + n^4 \mathrm{B}) \times 0{\cdot}00001,$$

where A and B are given in the following table:—

$\delta=$	A	B	$\delta=$	A	B
°	″	″	°	″	″
0	+0·0	0·0	25	+1·4	43·8
5	+0·1	10·4	30	+2·3	46·2
10	+0·3	20·5	35	+4·2	42·8
15	+0·6	29·9	40	+6·0	29·9
20	+0·8	37·9	45	+8·4	0·0

14. *Refraction.*—We have now to examine what effect refraction has upon the curvature of a trail. In *Monthly Notices*, lvii. pp. 133, &c., it is shown that the effect of a refraction $\mu \,.\, \tan$ Z.D. on a star whose coordinates are (x, y) when the coordinates of the zenith on the plate (supposed extended in the same plane so as to include the zenith) are (X,Y) is to increase x and y by Δx and Δy where

$$\Delta x = \mathrm{T}(\mathrm{X} - x) \qquad \Delta y = \mathrm{T}(\mathrm{Y} - y)$$

and

$$\mathrm{T} = \frac{\mu(1 + x^2 + y^2)}{1 + \mathrm{X}x + \mathrm{Y}y}$$

(T being the same as $-t$ in the paper cited, p. 136).

Now $\mu = 57''$ approximately; hence for our plates the max. value of $\mu x^3 = 0''{\cdot}057$. There is no doubt we may reject μx^4; but μx^3, especially if multiplied by 2 or 3, might introduce errors. Fortunately third powers of x and y do not occur in the present investigation, for we find the curvature of a trail by measuring the dip of the middle below the ends. If the effect of refraction be

$$a_0 + a_1 x + a_2 x^2 + a_3 x^3 + a_4 x^4 +, \text{ \&c.},$$

then at one end $x = +\xi$, and at the other $x = -\xi$, so that for the mean of the two the effect is

$$a_0 + a_2 \xi^2 + a_4 \xi^4 +, \text{ \&c.},$$

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and for the middle where $x=0$ the effect is a_0. Thus the effect on the height of ends above middle is

$$a_2\xi^2 + a_4\xi^4,$$

and since we may certainly reject fourth powers we need only consider the term $a_2\xi^2$.

15. We assume that X and Y are not much greater than unity; *i.e.* that the plate is exposed at a Z.D. not much greater than 45°.

Then

$$\frac{1}{\mu}T = (1 + x^2 + y^2)(1 + Xx + Yy)^{-1}$$

$$= [1 - Xx - Yy + x^2(1 + X^2) + 2XYxy + y^2(1 + Y^2)],$$

and

$$\frac{1}{\mu}\Delta y = Y - y(1 + Y^2) - xXY + x^2(1 + X^2) + xy(X + 2XY) + y^2(1 + Y + Y^2).$$

In this we put from § 8

$$y = y_0 + \tfrac{1}{2}\tan\delta\, x^2,$$

and thus the coefficient of x^2 in $\frac{1}{\mu}\Delta y$ is

$$-\tfrac{1}{2}\tan\delta\,(1 + Y^2) + (1 + X^2) + y_0\tan\delta\,(1 + Y + Y^2).$$

The last term is equivalent to a term in x^3, since y_0 is liable to the same maximum value as x. And if $Y=1$, $\tan\delta=1$ we may thus have from this term as much as $3 \times 0''\cdot057 = 0''\cdot17$. But under most conditions either $\tan\delta$ or Y would be small. We can, in fact, always expose a plate tolerably near the zenith if δ may be as much as 45°; and hence we shall neglect the term $y_0\tan\delta\,(1 + Y + Y^2)$.

Thus the effect of refraction on curvature is expressed by

$$z = \mu[(1 + X^2) - \tfrac{1}{2}\tan\delta\,(1 + Y^2)]\xi^2.$$

16. Now if the plate centre be kept fixed as in Case B of § 11, and different stars allowed to trail across it, then X and Y remain the same and $\tan\delta$ varies from D − 5°·7 to D + 5°·7. Thus the difference in the values of z at the edges is

$$z_1 - z_2 = \frac{\mu}{2}(\tan\delta_2 - \tan\delta_1)(1 + Y^2)\xi^2$$

$$= \frac{\mu}{2}\sec^2 D \times 0\cdot2(1 + Y^2)\xi^2 \text{ approx.}$$

The factor $\sec^2 D(1 + Y^2)$ is not much greater than unity; and since the maximum value of $\mu \times 0\cdot1 \times \xi^2$ is $0''\cdot056$, we may in general neglect this difference. Hence

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If various trails be taken on a plate 11°·5 × 11°·5 *whose centre is kept fixed and exposed to a point within* 45° *of the zenith, the curvatures of all trails will be equally affected by refraction, if quantities under* 0″·1 *be neglected.*

17. If, however, the same star be used but the plate centre moved about so as to take different trails of it, then X and Y vary, but tan δ remains constant. In this case

$$z_1 - z_2 = \mu\xi^2[(X_1^2 - X_2^2) - \tfrac{1}{2} \tan \delta \, (Y_1^2 - Y_2^2)].$$

When X_1, X_2, and Y_1, Y_2 are not very different this will be small. If the plate be exposed within 45° of the zenith, X_1, X_2, Y_1 and Y_2 are all less than unity; and since $\mu\xi^2$ has the value 0″·57 at maximum, we are not liable to an error greater than this if we neglect the effect of refraction entirely. For some work this knowledge is sufficient. For instance, in his Memoir * on the determination of longitudes by photography, Captain Hills neglects all quantities less than 1″; so that for his work the effect of refraction on the trails, whether of the same star or of different stars, may be neglected entirely, provided the plates be exposed at a Z.D. not greater than 45°.

18. For more accurate work we may examine the above expression a little more closely. The coordinates X and Y are determined from the equations

$$X = -\tan h \sin q \sec (P - q) \qquad Y = \tan (P - q),$$

where

$$\tan q = \tan \lambda \cos h$$

and h is the hour angle of the plate centre, λ the colatitude.

Now we shall suppose the plate exposed within a couple of hours of the meridian either way, which gives four working hours. Then the maximum value of h is ·5; of h^2 ·25; and of h^4 ·06. Thus as an approximation we may take

$$q = \lambda - \tfrac{1}{4}h^2 \sin 2\lambda$$
$$X = -h \sin \lambda \sec (P - \lambda) - h^3 \{\tfrac{1}{3} - \tfrac{1}{2} \cos \lambda \cos P \sec (P - \lambda)\}$$
$$Y = \tan (P - \lambda) + \tfrac{1}{4}h^2 \sin 2\lambda \sec^2(P - \lambda) + \text{fourth powers.}$$

Thus, if we neglect h^4, we may write

$$X^2 = h^2 \sin^2\lambda \sec^2(P - \lambda)$$
$$Y^2 = \tan^2(P - \lambda) + \tfrac{1}{2}h^2 \sin 2\lambda \tan (P - \lambda) \sec^2(P - \lambda).$$

Now P—λ is the meridian Z.D. of the plate centre. But it is readily seen that we shall not make a sensible error if we substi-

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* *Memoirs of Royal Astronomical Society*, vol. liii. p. 117.

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tute for this $p-\lambda$, the meridian Z.D. of the star. Call this ζ. Then

$$Y_1^2 = \tan^2\zeta + \tfrac{1}{2}h_1^2 \sin 2\lambda \tan \zeta \sec^2\zeta$$

$$Y_2^2 = \tan^2\zeta + \tfrac{1}{2}h_2^2 \sin 2\lambda \tan \zeta \sec^2\zeta$$

$$\therefore \tfrac{1}{2} \tan \delta \, (Y_1^2 - Y_2^2) = \tfrac{1}{4}(h_1^2 - h_2^2) \sin 2\lambda \tan \delta \tan \zeta \sec^2\zeta.$$

Now for ordinary latitudes, such as those of Europe, the product $\tan \delta \tan \zeta$ is small. For stars near the equator δ is small, and for stars of declination 45°, ζ is small. The maximum value of $\tan \delta \tan \zeta$ subject to the condition

$$\delta + \zeta = 90^\circ - \lambda = \phi \text{ (the latitude)}$$

is $\tan^2\frac{1}{2}\phi$; the value of which is as follows :—

Latitude =	30°	40°	50°	60°
$\tan^2\frac{1}{2}\phi$	·07	·13	·22	·34

Also $h_1^2 - h_2^2$ is not greater than ·25. Hence we can neglect the terms depending on Y.

As regards $X_1^2 - X_2^2 = (h_1^2 - h_2^2) \sin^2\lambda \sec^2\zeta$, the maximum value of this term, when $h_2 = 0$ and $h_1^2 = {\cdot}25$, is about ·25, and the effect on the refraction is about 0″·14. We may fairly neglect such small quantities except for the most refined investigations.

Thus in this case also the trails are equally affected by refraction in all parts of the plate, if the exposures be confined to the two hours preceding and the two hours following the meridian and the star be between the equator and the zenith.

Conclusions.

(*a*) Let a trail of a star declination δ be taken on a plate of field 11°·5 × 11°·5, whose centre is in declination D; and let (X, Y) be the standard coordinates of the zenith on the plate. Then the equation to the trail in standard coordinates (ξ, η) expressed in circular measure, is

$$\begin{aligned}
\eta = {} & \tan(\delta - D) \\
& + \tfrac{1}{2} \tan \delta \,.\, \xi^2 \\
& - \tfrac{1}{8} (\tan \delta - \tan^3\delta)\, \xi^4 && \text{(max. value } 1''{\cdot}0\text{)} \\
& - \tfrac{1}{4} \tan^2\delta \,.\, \tan(\delta - D) \,.\, \xi^4 && \text{(max. value } 0''{\cdot}5\text{)} \\
& + \mu\, [(1 + X^2) - \tfrac{1}{2} \tan \delta\, (1 + Y^2)]\xi^2 && \text{(max. value about } 0''{\cdot}6\text{).}
\end{aligned}$$

(*b*) If the exposures be at Z.D. less than 45°, and within two hours of the meridian, and if we take the *difference* between trails and not a single trail only, then the fifth line, due to refraction, may always be neglected, even for measurements of considerable precision.

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(*c*) Under the conditions specified in (*b*) the third and fourth lines are quickly calculated by the table in § 13.

(*d*) If, further, the trails be of the same star, we need *only the fourth line*, as given by the little table of § 10.

(*e*) If we neglect quantities less than $1''{\cdot}0$, then the curvatures of all trails on a plate exposed near the meridian between $D=0^\circ$ and $D=45^\circ$ should be the same, unless there is optical distortion.

Remarks on the Paper by Professor W. Schur, together with determination of the Diameter and Polar Compression of the Planet Mars from Observations with the Repsold Heliometer of the Remeis Observatory, Bamberg, and with the Breslau Heliometer at the Observatory, Strassburg, in 1879. By Dr. Ernst Hartwig.

(*Communicated by the Secretaries.*)

In the March number of the *Monthly Notices* (p. 330) Professor Schur has communicated a series of heliometer measurements of the polar and equatorial diameters of the planet *Mars*, from which he deduces a polar compression of a fiftieth. In that discussion no reference is made to the probable errors of the results which are said to be of a greater weight than the earlier researches because an ocular reversing prism was used. Computing the mean errors for the single measures and for the results of one day I have found them (in spite of "images being steady") greater than they were in the measurements made by the same observer with the little Breslau heliometer at Strassburg in 1877. The mean error of a diameter reduced to mean distance of the planet *Mars* from the Sun is for the polar diameter $\pm0''{\cdot}112$, for the equatorial $\pm0''{\cdot}094$ (in mean distance Sun—Earth $\pm0''{\cdot}170$ and $\pm0''{\cdot}143$), therefore for the measured diameter $\pm0''{\cdot}23$ and $\pm0''{\cdot}19$, or for a distance of *Mars* the same as in 1877 $\pm0''{\cdot}454$ and $\pm0''{\cdot}378$, the corresponding mean errors in 1877 having been for the measures made by Dr. Schur with the Breslau heliometer $\pm0''{\cdot}208$ and $\pm0''{\cdot}207$.

The three days of 1899 give a mean difference of $0''{\cdot}170$ between both directions (polar and equatorial) for the diameter in the mean distance between Sun and Earth, and the mean error of it is $\pm0''{\cdot}128$, because the single difference has the mean error $\pm0''{\cdot}222$. The measures in 1896 are better, the mean error for the result of a day being (polar diameter) $\pm0''{\cdot}031$ and (equatorial diameter) $\pm0''{\cdot}064$ in mean distance of the Earth from the Sun, whence we find for the single difference the mean error $\pm0''{\cdot}071$, and for the mean, $0''{\cdot}205$, of the four differences the mean error $\pm0''{\cdot}036$. But the measures in 1899 in view of the great uncertainty do not prove that the compression is in conflict with